Problem 1. Identical interest games and potential games

Consider an N-player game, in which each player P_i has a set Γ_i of pure strategies. Let $\Gamma = \Gamma_1 \times \cdots \times \Gamma_N$. A game is said to be of *identical interests* if all players have the same cost function, that is

$$J_i(\gamma) = \phi(\gamma) \quad \forall i = 1, \dots, N, \ \forall \gamma \in \Gamma.$$

a) Show that an identical interest game has a pure strategy Nash equilibrium. Furthermore, this Nash equilibrium is admissible.

b) Consider the identical interest game below. Show that it has two admissible Nash equilibria, but these equilibria are not interchangeable: $A = B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) Suppose γ is a Nash equilibrium for the game. Is it a minimizer of ϕ? Justify your answer.

Problem 2.

Provide two approaches for verifying whether the prisoner's dilemma game discussed in Lecture 01 is a potential game. Show your work for both approaches.

	Confess	Stay silent
Confess	$(5, 5)$	$(0, 10)$
Stay silent	$(10, 0)$	$(1, 1)$